Kalamunda Senior High

Year 12 Mathematics Methods - 2016

Unit 3 and Unit 4

Term	Time allocation (h)	Topic	Text Reference	Assessment
		Semester 1 – Unit 3		
1 & 2		**Assessment Items**	Investigation 1 Week 3 Value 5% Test 1 Week 5 Value 5% Test 2 Week 10 Value 7% Investigation 2 Week 12 Value 5% Test 3 Week 14 Value 8%	
1	7	**Exponential functions** 3.1.1 estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$, using technology, for various values of $a > 0$ 3.1.2 identify that e is the unique number a for which the above limit is 1 3.1.3 establish and use the formula $\frac{d}{dx}(e^x) = e^x$ 3.1.4 use exponential functions of the form Ae^{kx} and their derivatives to solve practical problems **Trigonometric functions** 3.1.5 establish the formulas $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ by graphical treatment, numerical estimations of the limits, and informal proofs based on geometric constructions 3.1.6 use trigonometric functions and their derivatives to solve practical problems	Chapter 1 Chapter 3 Chapter 4	
1	7+1	**Differentiation rules** 3.1.7 examine and use the product and quotient rules 3.1.8 examine the notion of composition of functions and use the chain rule for determining the derivatives of composite functions	Chapter 2 Chapter 3 Chapter 2 Chapter 4	Investigation 1 Week 3 Differentiation

		3.1.9 apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x\sin x$, $e^{-x}\sin x$ and $f(ax-b)$		
1	6+1	**The second derivative and applications of differentiation** 3.1.10 use the increments formula: $\delta y \approx \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x 3.1.11 apply the concept of the second derivative as the rate of change of the first derivative function 3.1.12 identify acceleration as the second derivative of position with respect to time 3.1.13 examine the concepts of concavity and points of inflection and their relationship with the second derivative 3.1.14 apply the second derivative test for determining local maxima and minima 3.1.15 sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection 3.1.16 solve optimisation problems from a wide variety of fields using first and second derivatives	Chapter 5 Chapter 6 Chapter 2 Chapter 10	Test 1 Week 5 Differentiation
1	7	**Anti-differentiation** 3.2.1 identify anti-differentiation as the reverse of differentiation 3.2.2 use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals 3.2.3 establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$ 3.2.4 establish and use the formula $\int e^x dx = e^x + c$ 3.2.5 establish and use the formulas $\int \sin x\, dx = -\cos x + c$ and $\int \cos x\, dx = \sin x + c$ 3.2.6 identify and use linearity of anti-differentiation 3.2.7 determine indefinite integrals of the form $\int f(ax-b)dx$ 3.2.8 identify families of curves with the same derivative function 3.2.9 determine $f(x)$, given $f'(x)$ and an initial condition	Chapter 7	

1	7	**Definite integrals** 3.2.10 examine the area problem and use sums of the form $\sum_i f(x_i)\ \delta x_i$ to estimate the area under the curve $y = f(x)$ 3.2.11 identify the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\ \delta x_i$ 3.2.12 interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$ 3.2.13 interpret $\int_a^b f(x)dx$ as a sum of signed areas 3.2.14 apply the additivity and linearity of definite integrals **Fundamental theorem** 3.2.15 examine the concept of the signed area function $F(x) = \int_a^x f(t)dt$ 3.2.16 apply the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically 3.2.17 develop the formula $\int_a^b f'(x)dx = f(b) - f(a)$ and use it to calculate definite integrals	Chapter 8 Chapter 9 Chapter 10	Test 2 Week 10 Differentiation and Anti-Differentiation
2	6+1	**Applications of integration** 3.2.18 calculate total change by integrating instantaneous or marginal rate of change 3.2.19 calculate the area under a curve 3.2.20 calculate the area between curves 3.2.21 determine displacement given velocity in linear motion problems 3.2.22 determine positions given linear acceleration and initial values of position and velocity.	Chapter 9 Chapter 10	
2	5+1	**General discrete random variables** 3.3.1 develop the concepts of a discrete random variable and its associated probability function, and their use in modelling data 3.3.2 use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable 3.3.3 identify uniform discrete random variables and use them to model random phenomena with equally likely outcomes 3.3.4 examine simple examples of non-uniform discrete random variables 3.3.5 identify the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases	Chapter 11	Investigation 2 Week 12 Discrete Random Variables

		3.3.6 identify the variance and standard deviation of a discrete random variable as measures of spread, and evaluate them using technology 3.3.7 examine the effects of linear changes of scale and origin on the mean and the standard deviation 3.3.8 use discrete random variables and associated probabilities to solve practical problems		
2	5+1	**Bernoulli distributions** 3.3.9 use a Bernoulli random variable as a model for two-outcome situations 3.3.10 identify contexts suitable for modelling by Bernoulli random variables 3.3.11 determine the mean p and variance $p(1-p)$of the Bernoulli distribution with parameter p 3.3.12 use Bernoulli random variables and associated probabilities to model data and solve practical problems	Chapter 12	
2	5	**Binomial distributions** 3.3.13 examine the concept of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial 3.3.14 identify contexts suitable for modelling by binomial random variables 3.3.15 determine and use the probabilities $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ associated with the binomial distribution with parameters n and p; note the mean np and variance $np(1-p)$ of a binomial distribution 3.3.16 use binomial distributions and associated probabilities to solve practical problems	Chapter 12	Test 3 Week 14 Integrals and discrete random variables
2		**Semester 1 EXAM**	Week 16 Value 10%	

Semester 2 – Unit 4				
Term	**Time allocation (h)**	**Topic**	**Text Reference**	**Assessment**
2 & 3		**Assessment Items**	Investigation 3 week 19 Value 5% Test 4 Week 21 Value 5% Test 5 Week 25 Value 7% Investigation 4 Week 27 Value 5% Test 6 Week 30 Value 8%	
2	9+1	**Logarithmic functions** 4.1.1 define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$ 4.1.2 establish and use the algebraic properties of logarithms 4.1.3 examine the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$ 4.1.4 interpret and use logarithmic scales 4.1.5 solve equations involving indices using logarithms 4.1.6 identify the qualitative features of the graph of $y = \log_a x$ $(a > 1)$, including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a (x - c)$ 4.1.7 solve simple equations involving logarithmic functions algebraically and graphically 4.1.8 identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems	Chapter 13	Investigation 3 Week 19 Integration
2/3	9+1	**Calculus of the natural logarithmic function** 4.1.9 define the natural logarithm $\ln x = \log_e x$ 4.1.10 examine and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$ 4.1.11 establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$	Chapter 14	Test 4 Week 21 Logarithmic Functiuon

		4.1.12 establish and use the formula $\int \frac{1}{x} dx = \ln x + c$, for $x > 0$ 4.1.13 determine derivatives of the form $\frac{d}{dx}(\ln f(x))$ and integrals of the form $\int \frac{f'(x)}{f(x)} dx$, for $f(x) > 0$ 4.1.14 use logarithmic functions and their derivatives to solve practical problems		
3	8	**General continuous random variables** 4.2.1 use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable 4.2.2 examine the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts 4.2.3 identify the expected value, variance and standard deviation of a continuous random variable and evaluate them using technology 4.2.4 examine the effects of linear changes of scale and origin on the mean and the standard deviation	Chapter 15	
3	7+1	**Normal distributions** 4.2.5 identify contexts, such as naturally occurring variation, that are suitable for modelling by normal random variables 4.2.6 identify features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution 4.2.7 calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems	Chapter 16 Chapter 17	Test 5 Week 25 Continuous random variables
3	6	**Random sampling** 4.3.1 examine the concept of a random sample	Chapter 18	

		4.3.2 discuss sources of bias in samples, and procedures to ensure randomness 4.3.3 use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli		
3	9+1	**Sample proportions** 4.3.4 examine the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ of the sample proportion $\hat{p}$ 4.3.5 examine the approximate normality of the distribution of $\hat{p}$ for large samples 4.3.6 simulate repeated random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$ where the closeness of the approximation depends on both n and p	Chapter 19	Investigation 4 Week 27 Chance and Data
3	7+1	**Confidence intervals for proportions** 4.3.7 examine the concept of an interval estimate for a parameter associated with a random variable 4.3.8 use the approximate confidence interval $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$ as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution 4.3.9 define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence 4.3.10 use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain p	Chapter 20	Test 6 Week 30 Interval estimates
4		**Semester 2 MOCK EXAM**	Week 32 Value 30%	

Assessment Information and Schedule

Assessment weightings	Tasks	Week/s	Task weighting	Assessment tasks and content descriptions
Unit 3				
Response 20%	**Test 1:** Differentiation	5	5%	Exponential and Trigonometric functions Differentiation rules
	Test 2: Differentiation	10	7%	The second derivative and applications of differentiation. Anti-differentiation
	Test 3: Integrals and discrete random variables	14	8%	Definite integrals and First Fundamental Theorem. Applications of integration. Discreet random variables
Investigation 10%	**Investigation 1:** Differentiation	3	5%	Calculus: An interesting derivative
	Investigation 2: Discrete random variables	12	5%	Probability distributions
Examination 10%	End of unit Examination	16	10%	Unit 3 content
	Semester 1 total		40%	
Unit 4				
Response 20%	**Test 4:** Logarithmic function	21	5%	Logarithmic functions. Calculus of natural logarithmic function
	Test 5: Continuous random variables	25	7%	General continuous random variables Normal distribution
	Test 6: Interval estimates	30	8%	Random sampling. Sample proportions
Investigation 10%	**Investigation 3:** Integration	19	5%	Calculus: A special integral $\int \frac{1}{x}\,dx = \ln x + c$ for $x > 0$ (Note: This is a common task with Mathematics Specialist outline)
	Investigation 4: Chance and Data	27	5%	Statistics: Simulate repeated random sampling
Examination 30%	Final Examination	32	30%	Unit 3 and 4 content
	Semester 2 total		60%	
		Year total	**100%**	

Structure of the syllabus

The Year 12 syllabus is divided into two units which are delivered as a pair. The notional time for the pair of units is 110 class contact hours.

Organisation of content

Unit 3

Contains the three topics:

- Further differentiation and applications
- Integrals
- Discrete random variables.

The study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

Unit 4

Contains the three topics:

- The logarithmic function
- Continuous random variables and the normal distribution
- Interval estimates for proportions.

The logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit, students are introduced to one of the most important parts of statistics, namely, statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Unit 3

Unit description

The study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. This supports the development of a framework for statistical inference.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit 4

Unit description

The calculus in this unit deals with derivatives of logarithmic functions. In probability and statistics, continuous random variables and their applications are introduced and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In the Mathematics Methods ATAR course, statistical inference is restricted to estimating proportions in two-outcome populations.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.